Engineering Road Note 8

January 2008

STATISTICALLY BASED QUALITY CONTROL FOR DENSITY IN ROAD CONSTRUCTION

1. INTRODUCTION

The potential of a material to perform adequately as part of a road structure is normally judged on the basis of measured values of certain physical properties, which are assumed to control that performance. These properties include the dry density ratio of compacted foundations, embankments, subgrades, sub bases and base courses and relative compaction (percent of Marshall Density) for asphalt courses.

Road building materials, whether natural or manufactured are not perfectly uniform. The values of their physical properties vary. While every effort is made in testing to be as precise as possible, there are limitations in equipment and in the test methods themselves, which contribute to testing variability. While every effort should be made to ensure uniformity of field compaction, there are limitations in construction equipment, which mean that every location in the road will not be subject to exactly the same process. This material variability, testing variability and process variability combine to form a total variability. It is this total variability, which is measured when a set of test measurements is made. It is not practical to separate variability into its components during routine quality control.

In the past, it was the practice to use what might be called the "traditional" form of specification in which density requirements were required to be "not less than" some nominated value. In theory that meant that all test results, and hence the work, had to meet the specified limit for compliance. In practice, it was considered that satisfactory work was achieved if an occasional result fell marginally below the specified level. The difficulty with this practice was that the extent and range of permitted departures from the specified requirement has been subject to individual interpretation. Then what the producer had to produce was not what was written in the specification.

To overcome these difficulties, the Main Roads Western Australia adopted a statistical form of specification, which recognises that a proportion of the works may have a density less than the nominated value. The basis, upon which this statistical form of specification was developed, is described in this Note.

2. FORM OF DENSITY SPECIFICATION

In the method of quality control described in this Note, density tests are carried out on an area of pavement termed a lot. A lot is a section of pavement considered for practical purposes to be homogeneous in terms of material quality and compaction process. A number of tests (n) are carried out and the mean ($\overline{x}$) and standard deviation(s) of the results are calculated. The specified requirement is expressed in the form:

$$R_c = (\overline{x} - ks) \geq L \qquad (1)$$

Where R_c = referred to as the characteristic dry density ratio for granular materials or percentage of Marshall density for asphalt

k = a multiplier

s = the standard deviation

L = the specification limit

Where $R_c \geq L$, the lot of work is accepted. Where $R_c < L$, there is a high probability that the work is of an unacceptable quality. In the case of unbound materials, the lot should be rejected and subject to reprocessing (see Section 11.0).

In the case of asphalt, reprocessing is not a practical requirement and the lot should only be rejected if it is grossly outside the acceptance limit. For lots, which are marginally outside, design requirements, the risk of reduced Performance is shared between the Consumer (the Main Roads) and the Producer (the Contractor) by the application of pay factors, which reduce payment to the Producer.

The form of specification defined by Equation 1 is referred to as an *"unknown variability"* quality control scheme. Other forms of quality control such as *"attribute"* and *"known variability"* schemes do exist [see Gray (1980)]. However, the *"unknown variability"* type is considered the most appropriate for the control of density requirements in pavements.

3. CALCULATION OF MEAN ($\overline{x}$) AND STANDARD DEVIATION (s)

The mean of a set of n test results from a lot is the average value and is given by: -

$$\overline{x} = \sum_{1}^{n} \frac{x_i}{n} \qquad (2)$$

Where x_i is an individual result.

The standard deviation of a set of n test results is a measure of the variability and is defined in AS 1057-1985 as: -

$$s = \sqrt{\frac{\sum_{1}^{n} (\overline{x} - x_i)^2}{n - 1}} \qquad (3)$$

4. SELECTION OF THE SPECIFICATION LIMIT (L)

The specification limit (L) is the nominal minimum dry density ratio (or percentage of Marshall density), which is considered necessary for adequate performance. It may be selected on the basis of strength tests or experience with similar materials.

In the case of natural gravel base courses, where California Bearing Ratio (CBR) tests have been carried out, the limit L should be selected to ensure a minimum four-day soaked CBR value of 80%. For subgrades, the limit L should correspond to the density at which California Bearing Ratio tests were carried out for pavement thickness design purposes.

Where laboratory test data is not available to the Designer, the values listed in the Main Roads Western Australia, Specifications 302, 501, 502, and 504 shall be used as applicable.

5. SELECTION OF THE MULTIPLIER (k)

The multiplier k is a function of the proportion defective (p), the producer's risk (α) and the number of tests per lot (n). A guide to the choice of appropriate values for each of these factors is presented below.

5.1 Selection of the Proportion Defective* (p)

A statistical form of specification recognises that a proportion of the works may have a density less than the nominated value. This is referred to as the *"proportion defective"*.

The value of the proportion defective (p) must be set such that the overall performance of the works is not adversely affected. This is to some extent an economic decision and is related to the initial cost of achieving a specified standard, and the long term maintenance cost. Current values used by Main Roads Western Australia are presented in Table 1.

TABLE 1
RECOMMENDED VALUES FOR THE PROPORTION DEFECTIVE (p)

TYPE OF FACILITY	PROPORTION DEFECTIVE (p)
Freeways	10%
Highways and Main Roads	15%
Other Roads	20%

* The proportion defective is also known as the fraction defective (AS1057-1985) and percentage defective (AS2490-1981).

5.2 Producer's Risk (α)

The "Producer's risk" (α) is the probability of the sampling and decision plan resulting in work of a satisfactory quality being rejected. This could occur if, by chance, all the randomly selected test sites for in situ density determination just happened to fall in areas of low density.

The Producer's risk (α) must be kept to a low value, but must be balanced against the risk of accepting unsatisfactory work (see Section 6.0) and the cost of testing. Where provision is made in the contract specification for "conditional acceptance" the *"Producer's risk"* is primarily one of reduced payment, rather than rejection, and a higher value of α is appropriate. The consumer also has an increased risk of accepting, albeit at a reduced price, work which is of a lower standard than that desired.

Current values of (α) used by Main Roads Western Australia are presented in Table 2.

TABLE 2

RECOMMENDED VALUES FOR PRODUCER'S RISK (α)

TYPE OF QUALITY CONTROL PLAN	**PRODUCER'S RISK (α)**
Simple Accept/Reject	10%
Provision for Conditional Acceptance	20%

5.3 Sample Size (n)

The cost of density testing of embankments and pavements is high. However, if too few tests are carried out, the Consumer's risk of accepting a sub-standard lot will reach an unacceptable level. The number of tests is therefore related to the relative significance in terms of future performance of the various components of the works. The number of tests per lot is presented in Table 3.

TABLE 3
RECOMMENDED NUMBER OF TESTS PER LOT (n)

WORKS COMPONENT	**NUMBER OF TESTS PER LOT (n)**
Embankment Foundations	6
Embankment Construction	6
Subgrade Preparation	6
Sub-Base	9
Base Course	9
Asphalt Surfacing	10

5.4 Determination of k for Known Values of p, α and n

Values of k for various combinations of p, α and n are presented in Table 4. The values shown in this table are based on the non central t distribution. Published tables of this distribution are fairly difficult to use and require interpolation. The values shown in Table 4 were calculated using a computer program developed by the Australian Road Research Board and described by Auff (1986).

For values not listed in the Table, an approximate solution, based on work by Jennet & Welch (1939), can be obtained using Tables for the standard normal distribution and solving Equation 4.

$$k_{\alpha} = \frac{k_p - k}{\left(\frac{1}{n} + \left(k^2 \div (2(n-1)\right)\right)^{\frac{1}{2}}} \qquad (4)$$

Where k_{α} = the standard normal variate corresponding to the Producer's risk (α)

k = the multiplier referred to in equation 1

k_p = the standard normal variate corresponding to the proportion defective (p)

Tables of the standard normal distribution appear in most statistics textbooks.

TABLE 4
VALUES OF THE MULTIPLIER (k)

p	α=5%			α=10%			α=20%			α=50%		
	10%	**15%**	**20%**	**10%**	**15%**	**20%**	**10%**	**15%**	**20%**	**10%**	**15%**	**20%**
n												
3	0.33	0.09	0.13	0.53	0.31	0.11	0.80	0.57	0.38	1.50	1.20	0.97
4	0.44	0.22	0.02	0.62	0.40	0.21	0.85	0.62	0.43	1.42	1.14	0.92
5	0.52	0.30	0.11	0.67	0.46	0.27	0.88	0.66	0.47	1.38	1.11	0.90
6	0.57	0.36	0.17	0.72	0.50	0.32	0.91	0.68	0.50	1.36	1.10	0.89
7	0.62	0.40	0.22	0.75	0.54	0.35	0.93	0.71	0.52	1.35	1.09	0.88
8	0.65	0.44	0.26	0.78	0.56	0.38	0.95	0.72	0.54	1.34	1.08	0.88
9	0.69	0.47	0.29	0.81	0.59	0.41	0.97	0.74	0.56	1.33	1.07	0.87
10	0.71	0.50	0.32	0.83	0.61	0.43	0.98	0.75	0.57	1.32	1.07	0.87
15	0.80	0.58	0.41	0.90	0.68	0.50	1.03	0.80	0.61	1.31	1.06	0.86
20	0.86	0.64	0.46	0.95	0.72	0.54	1.06	0.83	0.64	1.30	1.05	0.85

6. CONSUMER'S RISK

The *"Consumer's Risk"* (β) is the probability of the sampling and decision plan resulting in work of an unsatisfactory quality being accepted.

A number of approaches can be used to assess what proportion defective is likely to be associated with unsatisfactory performance. For natural gravel base courses, poor performance is associated with a four-day soaked CBR value less than 80%. For subgrades, a reduction in density, which leads to a 10% reduction in CBR, is likely to lead to a significant reduction in pavement life.

The approach recommended in this Note is based on the premise that the unsatisfactory product is one where the proportion defective (p) exceeds three times the acceptable level.

An approximate solution for β is defined by Equation 5.

$$k_\beta = \frac{k - k_p}{\left(\frac{1}{n} + \left(k^2 \div (2(n-1))\right)\right)^{\frac{1}{2}}} \qquad (5)$$

Where k_β = the standard normal variate corresponding to the Consumer's risk (β)

k = the multiplier referred to in equation 1

k_p = the standard normal variate corresponding to the proportion defective (p)

Example: Calculate the risk of accepting a lot of base course on a highway where n = 9, k = 0.59, p = 45%, k_p = 0.13.

$$k_\beta = \frac{0.59 - 0.13}{\left(\frac{1}{9} + \left(0.59^2 \div \left(2(9-1)\right)\right)\right)^{\frac{1}{2}}}$$

$$= 1.25$$

$$\beta = 10.5\% \text{ (from tables of the normal distribution)}$$

7. LOT SELECTION

The acceptance (or rejection) decision applies to the complete lot. If the characteristic value exceeds the specified limit, the whole of the lot is accepted even though some individual results are outside the limit. Likewise, if the characteristic value is below the limit, the entire lot is rejected.

Lot selection is the responsibility of the Consumer, for it is the Consumer who will be subjected to increased maintenance costs if defective product is accepted as a consequence of poor lot selection. Under Main Roads Western Australia contract works, Specification 201 vests the initial decision on lot selection with the Producer. However it is the Superintendent or other delegated personnel, who must ensure that the lot defined by the Producer satisfies specified criteria such as:

- The lot must be defined before testing commences,
- The lot may be of any size, but is restricted to a single day's production,
- The lot must be homogeneous with only random variation in characteristics such as density, moisture content, thickness, colour and finish.

Effectively the Consumer (Superintendent) must reserve the right to reject the lot if it is believed that the lot is not homogeneous with respect to any attribute. The literal meaning of homogeneous is "having the same origin". When applying the term to quality control of earthworks and pavement we add an additional requirement that the works process must be reasonably similar throughout the lot.

8. RANDOM SAMPLING

Every test or sample site within a lot must be selected randomly. The introduction of personal bias in the test or sample site selection process will invalidate the use of a statistically based quality control scheme.

Random sampling implies that every location within a lot has an equal chance of selection. To avoid problems of clustering of test or sampling points to one end of side of the lot, it is preferable to use stratified random sampling plans. With this technique the lot is divided into equal sized sub-lots equal to the number of required test or sample sites. One location is then selected at random from within each sub-lot.

9. RETESTING

The quality control scheme, described in this Note, does not provide for retesting for rejected lots. Certainly a lot, which just fails, may pass if it is retested. However, an unsatisfactory lot, which just passes, may fail if retested. Therefore there is no more justification for retesting a failed lot than there is for retesting a passed lot.

Retesting is only appropriate if there has been a mistake in a test procedure or if a result is manifestly impossible. It is generally sound practice to remove a result, which is more than two and a half standard deviations from the mean. If a result is removed, the multiplier (k) must be changed or a repeat test carried out. If it is necessary to remove more than one test result repeat testing should be conducted.

10. SHAPE OF DISTRIBUTION

There is a considerable amount of evidence that the distribution of dry density ratio test results can be approximated by the *"normal"* distribution. The normal distribution is a theoretical distribution which is bell shaped and symmetrical (see Figure 1).

The use of the normal distribution for the design of quality control plans can be likened to the use of linear elastic theory by structural engineers for the analysis of loads in buildings and bridges. Real materials such as steel are not perfectly linearly elastic but elasticity is a suitable approximation for practical design.

The real distributions of dry density ratio test results differ from the normal distribution in two ways: -

- The normal distribution has no upper or lower bounds. There are physical upper and lower bounds on density. For compacted natural gravel base course dry density ratios below 85% and above 105% are comparatively rare and values below 80% and above 115% should normally be discarded as "manifestly impossible".

- The normal distribution has zero skewness. Real Distributions may show positive or negative skewness. A negative skewness is more likely if the required standard of dry density is high, i.e. close to the practical upper bound.

Such departures from normality do not have a significant effect on the levels of Producer's and Consumer's risk. This is due to a fundamental mathematical fact that the distribution of means of samples from non-normal distributions tends towards normality (Central Limit Theorem). The distribution on characteristic values ($\bar{x} - ks$) do not, in the strictest mathematical sense, tend towards true normality. However in practical terms, the difference in distributions of values of ($\bar{x} - ks$) drawn from simple mono-modal (single peaked) non-normal distributions of x are not significantly different from distributions of ($\bar{x} - ks$) drawn from a normal distribution of x with the same mean and standard deviation.

At fig. 2 the distribution of 1000 characteristic values ($\bar{x} - ks$, n=9) of samples selected at random from a right triangular distribution is shown together with the distribution of 1000 characteristic values of samples selected at random from a normal distribution with the same mean and standard deviation. In the study that led to the development of this figure, it was found that for proportions defective in the range of interest (10% to 20%), the difference in probability of acceptance for these two distributions was less than 2% (see Figure 3).

It should be remembered in this context that there is nothing intrinsic about the values of 10% or 20% for Producer's risk. Provided decisions are based on the mean and the standard deviation rather than individual values, then the design of quality control schemes based on the assumption of normality of the parent population is sufficiently accurate.

11. REPROCESSING AND QUALIFIED REJECTION

At Section 7 it was pointed out that rejected lots should not, except in special circumstances, be retested before reprocessing. For reprocessing to be of an acceptable level it must be of such a nature that significant change in the distribution of dry density ratio test results can be expected. In many cases this will mean that the lot will have to be completely ripped, watered and re-compacted. However, circumstances can exist where a section is only marginally below the rejection limit and the moisture content is within 95% to 105% of Optimum Moisture Content (i.e. the lot is not over-wet or has dried out) when further rolling, without ripping up, can be expected to lead to a significant improvement in density.

The term "qualified rejection" is applied to lots where retesting is allowed after re-rolling without complete reprocessing. Qualified rejection should only apply to the first submission of a lot for testing. If a lot does not respond to re-rolling on the first occasion then it is unlikely to on subsequent occasions. Qualified rejection should not be allowed in the case of cement stabilised materials as re-rolling after curing may breakdown cementing bonds. As a guide "qualified rejection" should be limited to lots for which the characteristic density is within 1.0% of the acceptance limit (L).

12. OPERATING CHARACTERISTIC CURVES

The Producer's risk (α) used in selecting values of the multiplier (k) corresponds to a particular proportion defective (p). The Producer may increase the probability of his work being accepted by reducing the proportion defective. Conversely, any increase in proportion defective will increase the probability of rejection. The relationship between the probability of acceptance and the proportion defective for a particular quality control scheme is referred to as its operating characteristic (OC) curve.

An approximation to the OC curve can be obtained by using a range of values of p and hence k_p in conjunction with equation 5. To obtain a more exact solution is somewhat complicated and involves the use of the non-central t distribution and the reader is referred to the paper by *Auff* (1986) for more details.

OC curves for some recommended quality control schemes are included in Appendix 1 of this Note. These were calculated using a computer program developed by the Australian Road Research Board.

13. CONCLUDING REMARKS

No testing scheme produces perfect answers. The inherent variability of density ratio test results means that there is always the risk of a wrong acceptance/rejection decision. However, proper statistically based quality control plans recognises this variability and result in efficient use of available data such that the probability of a correct decision being reached is maximised.

REFERENCES:

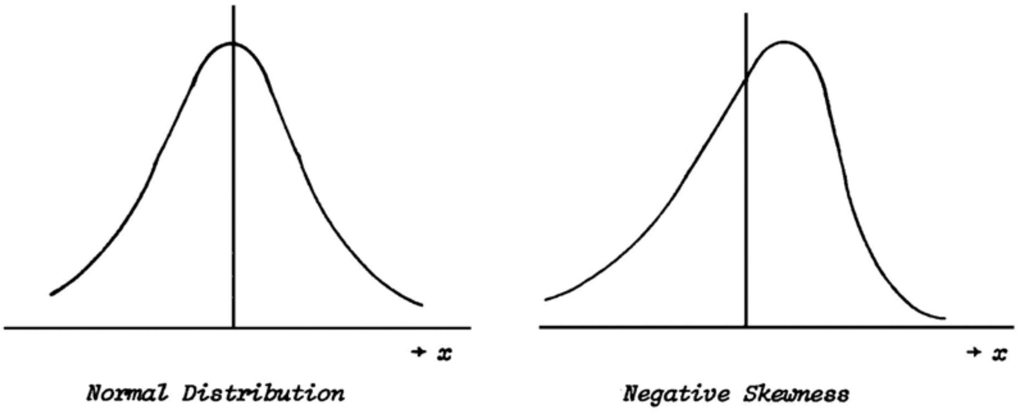
AUFF, A (1986): "The Selection of Statistical Compliance Schemes for Construction Quality Control". Special Report 30, Australian Road Research Board.

AUSTRALIAN STANDARDS ASSOCIATION (1981): "Sampling Procedures and Charts for Inspection by Variables for Percent Defective" AS 2490 SAA Sydney.

COCKS, G C (1984): "Effect of Non Normality of Producer's Risk with an Unknown Variability Scheme for Quality Control of Earthworks and Pavement Density" MRWA Materials Division ITN 2/84.

GRAY, W J (1980): "A Philosophy of Quality Control" Proc 2nd ARRB Workshop on Quality Control, Sydney.

JENNET, W J & WELCH, B L (1939): "The Control of Proportion Defective as Judged by a Single Quality Characteristic Varying on a Continuing Scale" Royal Statistical Society. Journal Supplement V6 pp 80-88.



Normal Distribution

Negative Skewness

FIGURE 1

NORMAL AND SKEWED DISTRIBUTIONS OF x

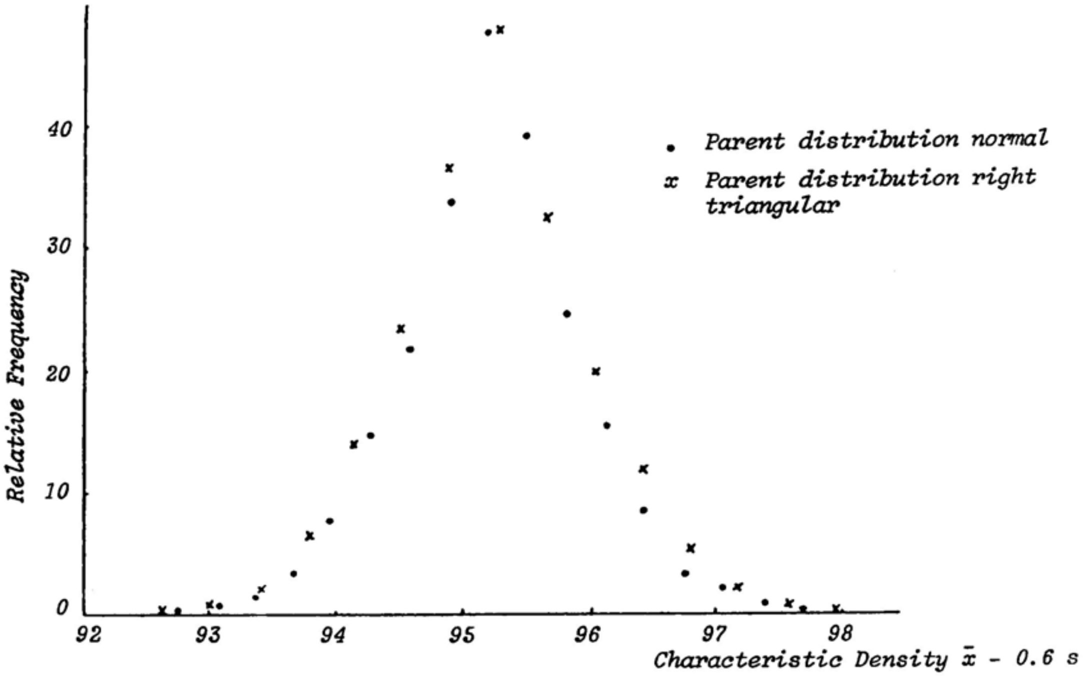
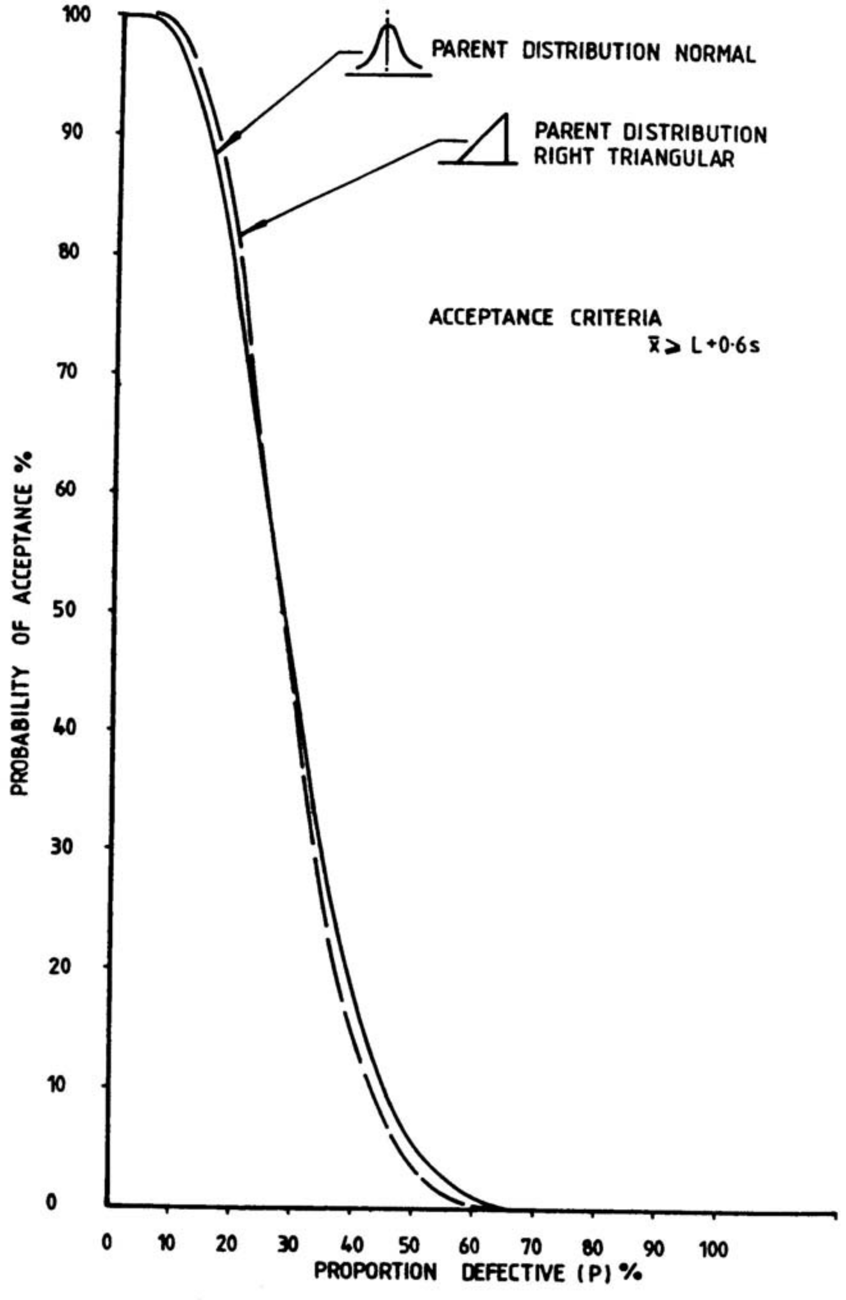
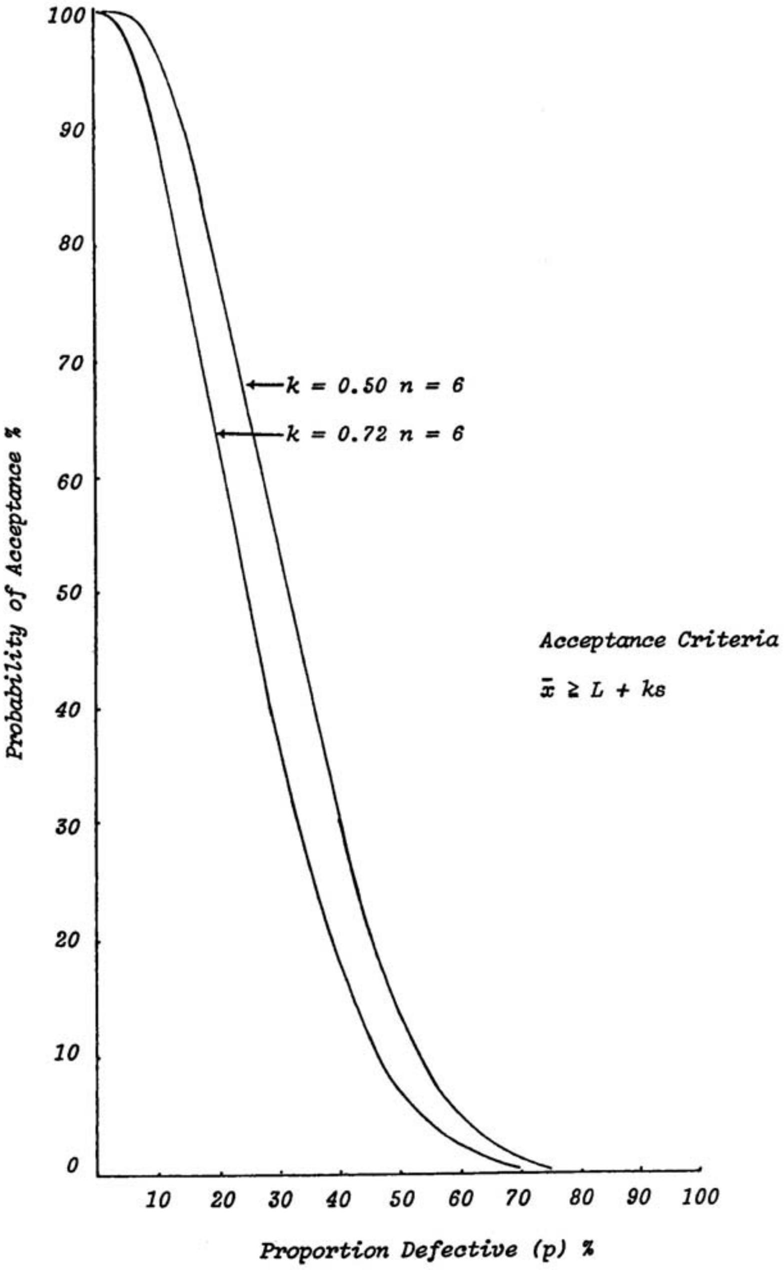


FIGURE 2

DISTRIBUTION OF 1000 VALUES OF $\bar{x}$ - 0.6 s (n = 9)
SELECTED AT RANDOM



OPERATING CHARACTERISTIC CURVES FOR UNKNOWN VARIABILITY SCHEME (n=9 , k=0·6)



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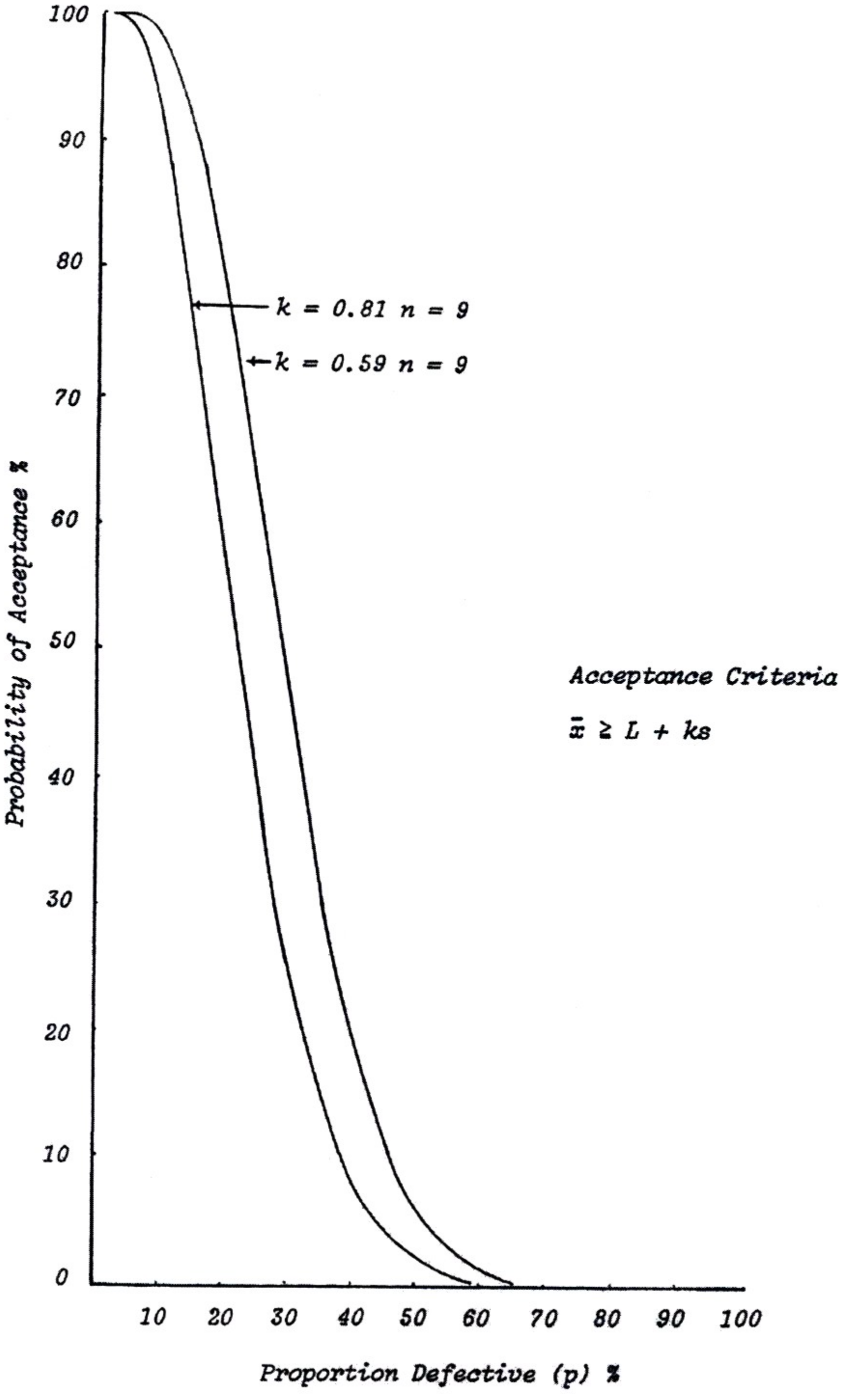
FOR UNKNOWN VARIABILITY SCHEME (n = 6)



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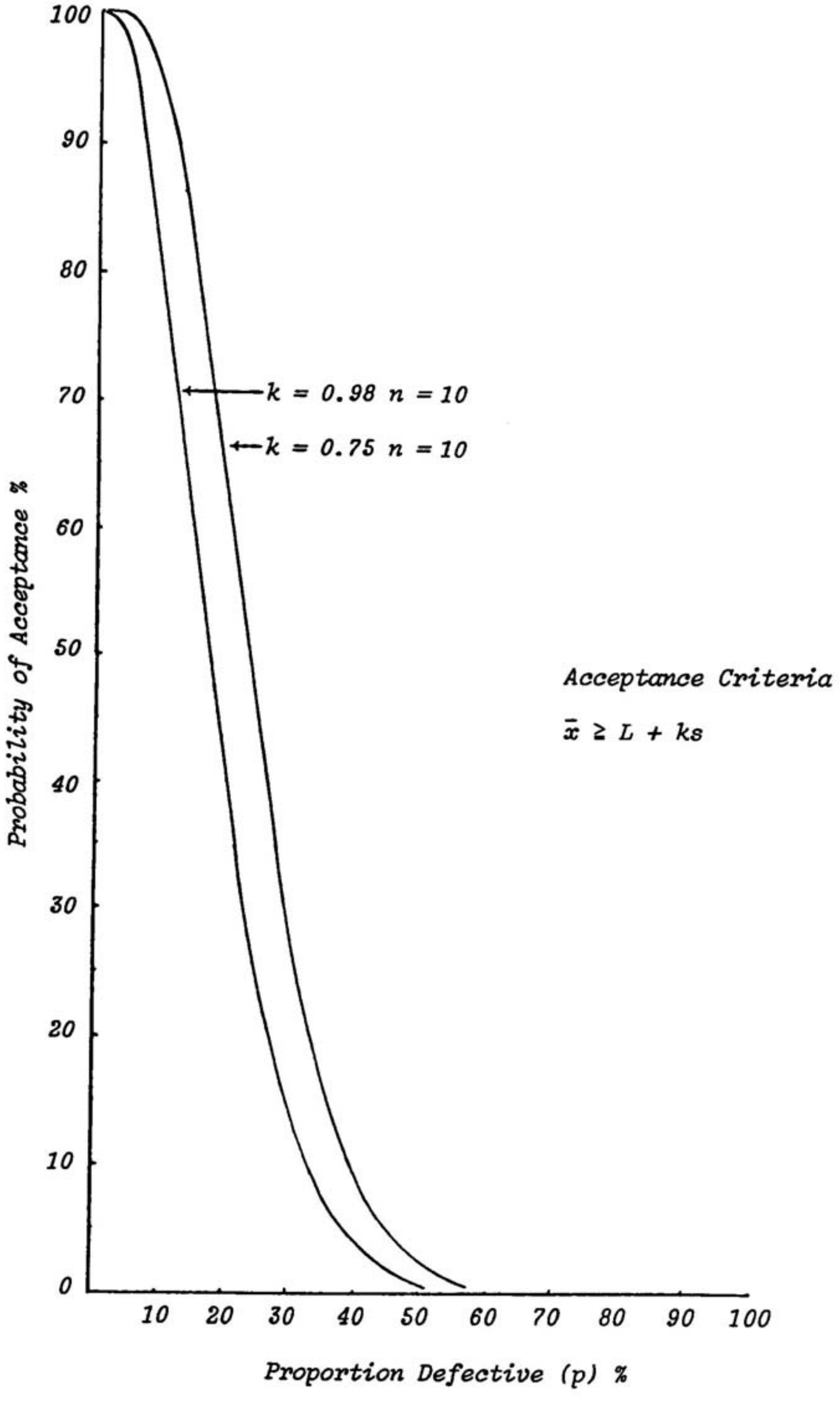

OPERATING CHARACTERISTIC CURVES
FOR UNKNOWN VARIABILITY SCHEME (n = 10)